Another proof of the infinitude of primes

Suppose there are only finitely many primes. Observe that 17 is prime, and let n be the product of all primes except 17.

Let S be the set of positive integers that are coprime to n, and let T be the set of positive integers that are 1 more than a multiple of n. Then $T \subseteq S$.

Now observe that we have

$$S = \{1, 17, 17^2, 17^3, \ldots\},$$

where the differences between consecutive elements are strictly increasing, and

$$T = \{1, n+1, 2n+1, 3n+1, \ldots\},$$

which contains infinitely many pairs whose difference is n. Hence T cannot be a subset of S. This contradiction shows that the number of primes cannot be finite.